(Pages : 4) **M – 1456**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS – I

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are compulsory. They carry **1** mark each.

1. State the axiom of completeness.
2. Give an example of infinite countable set.
3. State the Supremum property of R.
4. Find the limit points of the set $A = \left\{\frac{1}{n}: n \in N\right\}$.
5. Find $\lim_{n\to\infty} \frac{n}{2n+1}$.
6. What does it mean to say that a sequence (a_n) converges?
7. Give an example of a sequence which is bounded but not convergent.
8. Give an example of an open set which is not an interval.

9. Find the closure of $B = \left\{\frac{1}{3^n} n \in N\right\}$.

10. Give an example of a divergent sequence with converging subsequence.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. State the cut property of the real numbers.

12. Is the set of irrational real numbers countable? Justify.

13. Define a Cauchy sequence.

14. Show by an example that every bounded real sequence may not be convergent.

15. Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ for $n \geq 1$ converges to 2.

16. Check whether the sequence $\{n + (-1)^n\}$ is monotonic or not.

17. *Show that the sequence $\{(-1)^n\}$ is not convergent.*

18. Show that every convergent sequence is bounded.

19. Test for convergence the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$.

20. Discuss the convergence of $\sum \frac{1}{n^p}$.

21. Show that the series $\sum_{n=0}^{\infty} \frac{n}{n+1}$ diverges.

22. State the comparison test for convergence of series.

23. Show that the series $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ is divergent.

24. If the series $\sum_{k=1}^{\infty} a_k$ converges, then show that $\lim_{k \to \infty} a_k = 0$.

25. Discuss the convergence of geometric series.

26. State Baire's theorem

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

27. If $a, b \in R$ and $a < b$, show that there exists $r \in Q$ such that $a < r < b$.

28. Suppose that $\{a_n\}, \{b_n\}$ and $\{c_n\}$ are sequences such that $a_n \leq b_n \leq c_n$ for all $n \geq 10$, and that $\{a_n\}$ and $\{c_n\}$ both converge to 12. Then show that $\{b_n\}$ also converges to 12.

29. Define a Cantor set.

30. State and prove the Archimedean property of R.

31. Show that the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots \frac{n}{n+1}, \ldots\ldots$ is strictly monotone and bounded.

32. Show that sub sequences of a convergent sequence converge to the same limit as the original sequence.

33. Show that the union of arbitrary collection of open sets is open.

34. Show that the sequence $\left\{\frac{2n+3}{3n-2}\right\}$ is convergent.

35. Every convergent sequence is a Cauchy sequence.

36. Prove that Cauchy sequences are bounded.

37. Show that the series $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ is convergent.

38. Prove that A set $E \subset R$ is connected if and only if whenever $a < c < b$ with $a, b \in E$ then $c \in E$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

39. (a) Show that the open interval (0, 1) $=\{x \in R : 0 < x < 1\}$ is uncountable.

    (b) *State and prove nested interval property of real numbers*

40. (a) State and prove Cauchy Condensation Test.

    (b) Prove that every bounded infinite set has at least one limit point.

41. (a) Establish Cauchy criteria for convergence of sequence of real numbers.

    (b) If (a_n) and (b_n) are two sequences of real numbers converge to a and b respectively, then show that $lim(a_n + b_n) = lim\, a_n + lim\, b_n = a + b$.

42. (a) Let $x_1 = 2$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ for $n \geq 1$. Show that x_n convergence and find its limit.

    (b) If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then show that $\sum_{n=1}^{\infty} a_n$ converges as well. Is the converse true? Justify.

43. (a) *State and prove Monotone convergence theorem.*

    (b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

44. State and prove Heine - Borel theorem.

(2 × 15 = 30 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

39. (a) Show that the open interval (0, 1) $= \{x \in R : 0 < x < 1\}$ is uncountable.

    (b) State and prove nested interval property of real numbers

40. (a) State and prove Cauchy Condensation Test.

    (b) Prove that every bounded infinite set has at least one limit point.

41. (a) Establish Cauchy criteria for convergence of sequence of real numbers.

    (b) If (a_n) and (b_n) are two sequences of real numbers converge to a and b respectively, then show that $lim(a_n + b_n) = lim\, a_n + lim\, b_n = a + b$.

42. (a) Let $x_1 = 2$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ for $n \geq 1$. Show that x_n convergence and find its limit.

    (b) If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then show that $\sum_{n=1}^{\infty} a_n$ converges as well. Is the converse true? Justify.

43. (a) State and prove Monotone convergence theorem.

    (b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

44. State and prove Heine - Borel theorem.

(2 × 15 = 30 Marks)

(Pages : 4)

M – 1459

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Core Course VIII

MM 1544 – DIFFERENTIAL EQUATIONS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions :

1. Write the standard equation of linear differential equation.
2. Write the Lipschitz condition.
3. Solve $dy + y\,dx = 0$.
4. For what values of the constant m will $y = e^{mx}$ be the solution of $y'' - 3y' - 10y = 0$.
5. Check whether $y^2dy + x^2dx$ exact or not.
6. Find the complementary function of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^x \sin x$. .
7. Define Wronskian
8. Write the standard form of Legendre's linear equation

P.T.O.

9. Write the characteristic equation of $2\frac{d^2y}{dx^2}-\frac{dy}{dx}-3y=0$.

10. Define basis of solutions of a homogeneous second order differential equation

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions

11. Find the order and degree of the ODE $\frac{d^3y}{dx^3}+2\left(\frac{dy}{dx}\right)^{\frac{1}{2}}=0$

12. Define partial differential equation. Give one example of it

13. Solve $\frac{dy}{dx}=xy+x$.

14. State the uniqueness theorem of first order differential equation.

15. Verify that $y=\frac{2}{x}$ is a solution of the differential equation $xy'=-y$,for all $x\neq 0$.

16. Show that a seperable equation is also exact.

17. Check the exactness of $y'=1+y^2$.

18. Find the integrating factor of $y\,dx-x\,dy=0$.

19. Find the general solution of $\frac{d^2y}{dx^2}+4y=0$.

20. Find a differential equation whose solution is cos 3x.

21. Find the complementary function of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3e^x$.

22. Write the basis of solution of the equation $\frac{d^2y}{dx^2}+y=0$

23. Write the standard form of Euler- Cauchy equation. Give one example of it.

24. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2 = 0$

25. Find a general solution of $x^2y' - 20y = 0$.

26. Find the Wronskian of e^x and e^{-x}.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions

27. Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

28. Solve $(x + 4)(y^2 + 1)dx + y(x^2 + 3x + 2)dy = 0$

29. Find the Orthogonal Trajectories of the family $cx^2 + y^2 = 1$

30. Solve the initial value problem $y' + y\tan x = \sin 2x, y(0) = 1$

31. Solve $x\frac{dy}{dx} + y = xy^{\frac{3}{2}}, y(1) = 4$.

32. Solve $(x^2 - 3y^2)dx + 2xy\,dy = 0$

33. By reducing the order, solve $(x^2 + 1)y'' - 2xy' + 2y = 0$, given x is one solution

34. Solve $\frac{d^2y}{dx^2} + y = \sin x$.

35. Find the general solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$

36. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 16y = 0$.

37. Solve the logistic equation $y' = Ay - By^2$.

38. Solve $y'' + y = \operatorname{cosec} x$ using the method of variation of parameters.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions

39. (a) Solve $\left(\frac{3-y}{x^2}\right)dx+\left(\frac{y^2-2x}{xy^2}\right)dy=0$, $y(-1)=2$ by exactness.

(b) Find an integrating factor and solve

$(5xy+4y^2+1)dx+(x^2+2xy)dy=0$

40. (a) Solve the initial value problem $(ye^x+2e^x+y^2)dx+(e^x+2xy)dy=0$, $y(0)=6$

(b) Find a basis of solutions of the differential equation $(x^2-x)y''-xy'+y=0$.

41. (a) Check the exactness and solve $(2xy^2+y)dx+(2y^3-x)dy=0$.

(b) Solve the initial value problem $(y+\sqrt{x^2+y^2})dx-xdy=0$, $y(1)=0$.

42. (a) Solve $x^2y''-2xy+2y=0$, $y(1)=1$, $y'(1)=1$.

(b) Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2x^2+e^x+2xe^x+4e^{3x}$.

43. (a) Solve $(D^2+2D+\frac{3}{4}I)y=3e^x+\frac{9}{2}x$.

(b) Solve $y'''-3y''+2y'=0$.

44. (a) Solve the initial value problem

$y''-2y'-3y=2e^x-10\sin x$, $y(0)=2$, $y'(0)=4$

(b) Solve $(D^2+3D+2I)y=5x^2$.

(2 × 15 = 30 Marks)

(Pages : 4) **M – 1458**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course VII

MM 1543 – ABSTRACT ALGEBRA – GROUP THEORY

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

(**All** questions are compulsory. These questions carry **1** mark each)

1. Define an associative binary operation.

2. Let a and b belong to a group G. Find an x in G such that $xabx^{-1} = ba$.

3. Define the centre of a group.

4. Find μ^{100} if $\mu = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{bmatrix}$.

5. Find the order of the permutation $(23)(156)$.

6. Find $Aut(z)$.

7. Define normal subgroup.

P.T.O.

8. What is the order of the factor group $\frac{z60}{<15>}$.

9. Find the Kernel of the mapping $\varphi : R^* \to R^*$ defined by $\varphi(x) = |x|$.

10. Find the left cosets of $H\{0, \ In, \ Iz, ...\}$ in Z where n is a positive integer.

SECTION – II

(Answer any **eight** questions. These questions carry **2** marks each.)

11. Prove that the left and right cancellation laws hold in a group.

12. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.

13. Prove that for each a in a group G, the centralizer of a is a subgroup of G.

14. Find all generators of z_{10} and z_{12}.

15. Prove that every cyclic group is abelian.

16. Express $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{bmatrix}$ as a product of cycles.

17. Prove that for $n > 1$, A_n has order $\frac{n!}{2}$.

18. Let $\varphi : G \to \overline{G}$ is an isomorphism. The prove that G is abelian if and only if $\overline{G}$ is abelian.

19. Show that z has infinitely many subgroups isomorphic to z.

20. Let H be a subgroup of G. Then prove that $aH = bH$ if and only if $a^{-1}b \in H$.

21. Let G be a group and $a \in G$. Show that $a^{|G|} = e$.

22. Let $|a| = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there? List them.

23. Prove that the centre $Z(G)$ of a group G is normal.

24. Prove that a factor group of an abelian group is abelian.

25. Prove that a normal subgroup N is the Kernel of the mapping $g \to gN$ from G to G/N.

26. Prove that the mapping $\varphi : GL(Z, R) \mapsto R^*$ defined by $\varphi(A) = \det A$ is a homomorphism.

SECTION – III

(Answer any **six** questions. These questions carry **4** marks each.)

27. Show that if G is a finite group with even number of elements, then there is an $a \neq e$ in G such that $a^2 = e$.

28. Prove that the set of all 2×2 matrices with entries from R and determinant as '1' is a group under matrix multiplication.

29. Prove that in a group, an element and its inverse have the same order.

30. For every integer $n > 2$, prove that the group $u(n^2 - 1)$ is not cyclic.

31. Show that every permutation on a finite set can be written as a cycle or as a product of cycles.

32. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$. Write α, β and $\alpha\beta$ as product of disjoint cycles.

33. Prove that for every positive integer n, $Aut(Z_n)$ is isomorphic to $u(n)$.

34. State and prove Fermat's little theorem.

35. Let H be a normal subgroup of a group G and K be any subgroup of G. Then $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G.

36. Let G be a group and $Z(G)$ be the centre of G. Prove that if $G/Z_{(G)}$ is cyclic, then G is abelian.

37. Let $\varphi : G \to \overline{G}$ be a group homomorphism and let $g \in G$. Prove that if $\varphi(g) = g'$, then $\varphi^{-1}(g') = \{x \in G \mid \varphi(x) = g'\} = gKer\varphi$.

38. Find all abelian groups of order 360, upto isomorphism.

SECTION – IV

(Answer any **two** questions. These questions carry **15** marks each)

39. (a) Let * be defined on Q^+ by $a * b = \frac{ab}{4}$. Prove that $(Q,*)$ is an abelian group.

    (b) Prove that if a and b are elements of a group G, then the linear equations $ax = b$ and $ya = b$ have unique solutions x and y in G.

40. (a) Show that a nonempty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, for all $a, b \in H$.

    (b) Let a be an element of order n in a group and let k be a positive integer. Prove that $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$ and $|a^k| = n / \gcd(n,k)$.

41. (a) Prove that the collection of all permutations of a finite set is group under permutation multiplication.

    (b) If the pair of cycles $\alpha = (a_1, a_2, \ldots. a_m)$ and $\beta = (b_1, b_2, \ldots. b_n)$ have no entries in common, then $\alpha\beta = \beta\alpha$.

42. Suppose that $\varphi : G \to \overline{G}$ is a group isomorphism. Prove that

    (a) For every integer n and for every a in $G, \varphi(a^n) = [\varphi(a)]^n$.

    (b) $G = \langle a \rangle$ if and only if $\overline{G} = \langle \varphi(a) \rangle$.

    (c) φ carries the identity of G into the identify of $\overline{G}$.

43. (a) State and prove Lagrange's theorem.

    (b) Is the converse of Lagrange's theorem true? Justify.

44. Let $\varphi : G \to \overline{G}$ be a group homomorphism and let H be a subgroup of G. Prove that

    (a) If H is normal in G, then $\varphi(H)$ is normal in G.

    (b) If $|H| = n$, then $|\varphi(H)|$ divides n.

    (c) If $\overline{K}$ is a subgroup of $\overline{G}$, then $\varphi^{-1}(\overline{K}) = \{K \in G | \varphi(K) \in \overline{K}\}$ is a subgroup of G.

(Pages : 6) **M – 1462**

Reg. No. : ....

Name : ....

Fifth Semester B.A./B.Sc./B.Com. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Open Course

MM 1551.3 – BASIC MATHEMATICS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

I. Answer **all ten** questions. These questions carry **1** mark each

1. Determine the place value of 2 in 417,216,900.

2. Find 45+57.

3. State commutative property of addition.

4. Identify the numerator and denominator of the fraction $\frac{3}{5}$.

5. Define Least Common Multiple (LCM).

6. What is a decimal fraction?

7. Round 4.81542 to the thousandths place.

8. In an algebra class there are 15 women and 17 men. Write the ratio of women to men.

9. What is pictograph?

10. What is a circle graph?

(10 × 1 = 10 Marks)

II. Answer **any eight** questions from among questions **11 to 26**. These questions carry **2** marks each

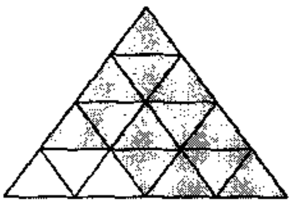
11. Simplify the expression 48 ÷ 8. Identify the dividend, divisor, and quotient.

12. State any two properties of division.

13. Evaluate 5^3.

14. Simplify $36 \div (7^2 - 3)$.

15. Write a fraction for the shaded portion and for the unshaded portion of the following figure



16. Convert the mixed number $7\frac{1}{4}$ to an improper fraction.

17. Simplify $\frac{1}{3} + \frac{3}{5} \div \frac{9}{10}$

18. Find the LCM of 10, 15 and 8.

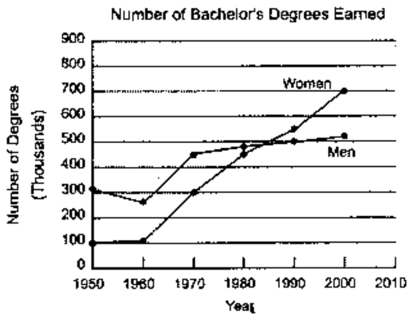
19. Jane Marie bought 8 cans of tennis balls for \$1.98 each. She paid \$1.03 in tax. What was the total bill?

20. Find $30.55 \div 13$.

21. The town of Roxbury, Connecticut, had 1825 people in the year 1990. By the year 2008, the U.S. Census Bureau projects its population to be 2441. Write a ratio depicting the increase in population to the number of people in the town in 1990.

22. A health club charges $125 for 20 visits. Find the unit rate in dollars per visit.

23. The following figure shows the number of bachelor's degrees earned by men and women for selected years.



(a) In 1950, who earned more bachelor's degrees, men or women?

(b) In 2000, who earned more bachelor's degrees, men or women?

24. A small business employs five workers. Their yearly salaries are $42,000 $36,000 $45,000 $35,000 $38,000. Find the mean yearly salary for the five employees.

25. What is the procedure to find the median of a list of numbers?

26. Find the probability of rolling a 5 or greater on a die.

(8 × 2 = 16 Marks)

III. Answer **any six** questions from among questions **27 to 38**. These questions carry **4** marks each

27. Write the four steps of the order in which operations are to be performed?

28. A 5-speed Jeep Cherokee gets 23 mpg (miles per gallon) on the highway. How many gallons of gas would be required for a 667-mi drive from El Paso to Dallas?

29. Divide $\frac{82705}{602}$.

30. The population of Texas comprises roughly of the population of the United States. If the U.S. population is approximately 296,000,000, approximate the population of Texas.

31. Simplify $\left(\frac{3}{5} \div \frac{2}{15}\right)^2$.

32. Find $1\frac{2}{3} \div 6 \cdot \left(\frac{3}{10}\right)$.

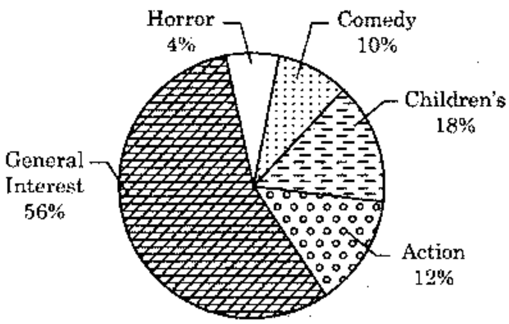
33. The Mona Lisa is perhaps the most famous painting in the world. It was painted by Leonardo da Vinci somewhere between 1503 and 1506 and now hangs in the Louvre in Paris, France. The dimensions of the painting are 30 in. by 20.875 in. What is the total area?

34. Round $45.\overline{45}$ to the hundredths place. Then use the rounded value to estimate whether the product $45.\overline{45} \times 1.1$ is close to 50.

35. Solve the proportion $\frac{0.8}{3.1} = \frac{4}{p}$.

36. On a sunny day, a 6-ft man casts a 3.2-ft shadow on the ground. At the same time, a building casts an 80-ft shadow. How tall is the building?

37. A certain video rental store carries 2000 different videos. It groups its video collection by the categories shown in the following graph.



(a) How many videos are comedy?

(b) How many videos are action or horror?

38. Find the indicated probability.

(a) The probability of getting a winter cold is $\frac{3}{10}$. What is the probability of not getting a winter cold?

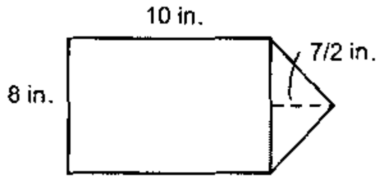
(b) If the probability that a washing machine will break before the end of the warranty period is 0.0042, what is the probability that a washing machine will not break before the end of the warranty period?

(6 × 4 = 24 Marks)

IV. Answer **any two** questions from among questions **39 to 44**. These questions carry **15** marks each

39. (a) Linda must drive from Clayton to Oakley. She can travel directly from Clayton to Oakley on a mountain road, but will only average 40 mph. On the route through Pearson, she travels on highways and can average 60 mph. Which route will take less time?

(b) Find the total area of the following figure.



40. Carson estimates that his total cost for college for 1 year is $12,600. He has financial aid to pay of the cost.

(a) How much money is the financial aid worth?

(b) How much money will Carson have to pay?

(c) If Carson's parents help him by paying of the amount not paid by financial aid, how much money will be paid by Carson's parents?

41. Jason and Sara plan to paint a side of their house whose dimensions are given in the following figure.

(a) How much area will they have to paint?

(b) They want to string Christmas lights around the triangular portion of the house. What length is required for the string of lights?

42. (a) A negative for a photograph is 3.5 cm by 2.5 cm. If the width of the resulting picture is 4 in., what is the length of the picture?

(b) If a cable 25 ft long weighs 1.2 lb. how much will a 120-ft cable weigh?

(c) Convert $\frac{21}{31}$ to decimal form rounded off to hundredth place.

43. Explain each of the following with suitable examples and diagram.

(a) Bar graph

(b) Picto graph

(c) Line graph

44. Use the Gaussian elimination method to find x, y and z where

$$2x - y + 3z = 5$$
$$-4x - 2y - 3x = 8$$
$$3x + y - z = 4$$

(2 × 15 = 30 Marks)

(Pages : 6) **M – 1457**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Core Course VI

MM 1542 COMPLEX ANALYSIS I

(2018 & 19 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

(Answer the **ten** questions are compulsory. They carry **1** mark each)

1. Find the quotient $\frac{5}{(1-i)(2-i)(3-i)}$.

2. Find the conjugate of $\frac{1+2i}{1-(1-i)^2}$.

3. Find the argument of $\left(\sqrt{3}-i\right)^2$.

4. Let $S=\left\{1,\frac{1}{2},\frac{1}{3},....\right\}$ what is the boundary of S.

5. Define an analytic function.

P.T.O.

6. Write the polynomial $z^4 - 16$ is factored form.

7. Find $\log(-1)$.

8. Find the principal value of i^{2i}.

9. Compute $\int_0^1 (2t + it^2)dt$.

10. State Cauchy's integral theorem.

SECTION – II

(Answer any **eight** questions. These questions carry **2** marks each.)

11. Find z if $z^2 - 2z - 2 = 0$.

12. Evaluate $(1-i)^4$.

13. Find the absolute value of $\frac{(1+3i)(1-2i)}{3+4i}$.

14. Show that for all z, $e^{2+\pi i} = e^{-z}$.

15. Write $f(z) = \frac{z+i}{z^2+1}$ in the form $w = u(x, y) + i\ v(x, y)$.

16. Show that $f(z) = Imz$ is nowhere differentiable.

17. Find $\lim_{z \to 5} \frac{3z}{z^2 - (5-i)z - 5i}$.

18. Discuss the analyticity of the function $\frac{z}{\overline{z}+2}$.

19. Show that if v is a harmonic conjugate of u in a domain D, then uv is harmonic in D.

20. Write the polynomial $(z-1)(z-2)^3$ in the Taylor form, centred at $z=2$.

21. Show that $\tan z$ is periodic with period π.

22. Find the maximum value of $|z^2+3z-1|$ in the disk $|z|\le 1$.

23. Find all values of $(1+i)^i$.

24. Show that the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is a smooth curve by producing an admissible parametrization.

25. Evaluate $\int_\Gamma e^z dz$ along the upper half of the circle $|z|=1$ from $z=1$ to $z=-1$.

26. Compute the integral $\int_\Gamma \frac{e^z+\sin z}{z}\,dz$, where Γ is the circle $|z-2|=3$ traversed once in the counter clockwise direction.

SECTION – III

(Answer any **six** questions. These questions carry **4** marks each)

27. Find the complex numbers z_1 and z_2 that satisfy the system of equations

$$(1-i)z_1+3z_2=2-3i$$
$$iz_1+(1+2i)z_2=1$$

28. Write the quotient $\frac{1+i}{\sqrt{3}-i}$ in polar form.

29. Prove that $1+w_m+w_m^2+...+w_m^{m-1}=0$.

30. Suppose that $f(z)$ and $\overline{f(z)}$ are analytic in a domain D. Show that $f(z)$ is constant in D.

31. Find the partial fraction decomposition of the rational function $\dfrac{2z+i}{z^3+z}$.

32. Establish $\sin z_1 \cos z_2 + \sin z_2 \cos z_1 = \sin(z_1+z_2)$.

33. Determine a branch of $f(z)=\log(z^3-2)$ that is analytic at $z=0$ and find $f(0)$ and $f'(0)$.

34. Derive the identity $\tan^{-1} z = \dfrac{i}{2}\log\left(\dfrac{i+z}{i-z}\right)$.

35. Prove that if C is the circle $|z|=3$, traversed once, then $\left|\int_C \dfrac{dz}{z^2-i} \le \dfrac{3\pi}{4}\right|$.

36. Determine the possible values for $\int_\Gamma \dfrac{1}{z-a}dz$, where Γ is any circle not passing through $z=a$, traversed once in the counter clockwise direction.

37. Compute $\int_C \dfrac{\sin z}{z^2(z-4)}dz$ where C is the circle $|z|=2$ traversed once in the positive sense.

38. State and explain maximum modulus principle.

SECTION – IV

(Answer any **two** questions. These questions carry **15** marks each)

39. (a) Describe the set of points z in the complex plane that satisfies each of the following.

(i) $|2z - i| = 4$

(ii) $|z| = \text{Re}\, z + 2$

(iii) $|z - i| < 2$.

(b) Compute the integral $\int_0^{2\pi} \cos^4 \theta \, d\theta$ by using exponential form of $\cos\theta$ and binomial formula.

40. (a) Prove that if $f(z)$ is analytic in a domain D and if $f'(z) = 0$ everywhere in D, then $f(z)$ is constant in D.

(b) Prove that if $f(z) = u(x, y) + i\, v(x, y)$ is analytic in a domain D, then each of the functions $u(x, y)$ and $v(x, y)$ is harmonic in D. Construct an analytic function whose real part is $u(x, y) = x^3 - 3xy^2 + y$.

41. (a) Prove that $\sin z = 0$ if and only if $z = k\pi$, where k is an integer.

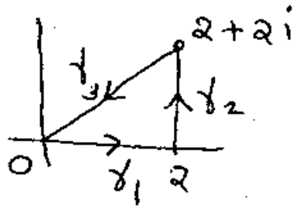
(b) Prove that the function e^z is one to one on any open disk of radius π.

(c) Find all numbers z such that $e^{iz} = 3$.

42. (a) Prove that the function $Log\ z$ in analytic in the domain D^* consisting of all point of the complex plane except those lying on the non positive real axis. Also $\frac{d}{dz} \log z = \frac{1}{z}$, for z in D^*.

(b) Find all the solutions of the equation $\cos z = 2i$.

43. (a) Compute $\int_{\Gamma} \bar{z}^2 dz$ along the simple closed contour Γ given below.



(b) Suppose that the function $f(z)$ is continuous in a domain D and has an antiderivative $F(z)$ throughout D. Prove that for any contour Γ lying in D, with initial point z_I and terminal point z_T, $\int_{\Gamma} f(z)dz = F(z_T) - F(z_I)$.

44. (a) State and prove Morera's theorem.

(b) State and prove fundamental theorem of Algebra.